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AUTHOR Souviney, Randall J.
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ABSTRACT

This set of 13 activities, adapted from "Mathmatters: Developing Computational Skills with Developmental Activity Sequences (by Randall Souviney, Tamara Keyser, and Alan Sarver) was designed to help teachers using the Indigenous Mathematics Project materials (in Papua New Guinea) understand the basic processes involved in computation with whole numbers and provide background in the use of appropriate concrete models. The activities focus on base ten materials, place value, addition, subtraction, multiplication, and division. Recording sheets for pupil work are included. (MNS)

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INDIGENOUS MATHEMATICS PROJECT

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LEARNING TO COMPUTE

Randall J. Souviney

Working Paper 1
January 1980

THE INDIGENOUS MATHEMATICS PROJECT

This series of working papers and research reports has been published to provide a comprehensive source of documentation on research and development efforts associated with the Indigenous Mathematics Project. The Project was established in 1977 by the Government of Papua New Guinea to investigate various aspects of traditional mathematics development. During the first phase (1977–79) of the five year program, basic research in crosscultural cognitive development was carried out and indigenous counting, classification and measurement systems used throughout the country were documented. During the second phase of the project (jointly funded by the Department of Education and UNESCO), pilot instructional materials were developed and trialled. The intent was to assess the feasibility of utilizing complementary aspects of indigenous and Western mathematics as a basis for developing culturally relevant student materials, instructional aids and teachers' guides which reflect the practical constraints of the community school environment. The results of IMP work are intended to inform future curriculum development in an effort to provide more appropriate instructional materials and learning aids for Community Schools throughout Papua New Guinea.

Series Editor: Randall Souviney
Managing Editor: Jacquie Wathen

LEARNING TO COMPUTE

Randall J. Souviney
UNESCO Advisor

Indigenous Mathematics Project
January 1980

This guide was prepared for use during an IMP in-service course in April 1980. The activities have been adapted from Mathmatters: Developing Computational Skills with Developmental Activity Sequences, Randall Souviney, Tamara Keyser and Alan Sarver, (Goodyear Publications, Inc.), 1977.

LEARNING TO COMPUTE

The following set of activities are designed to help teachers understand the basic processes involved in computation with whole numbers. Six sections are included:

	grades (approximate)
1. Base-10 materials	1 - 6
2. Place-Value.	1 - 3
3. Addition.	2 - 4
4. Subtraction.	3 - 5
5. Multiplication.	4 - 6
6. Division.	5 - 6

The set of 13 activities is not intended as a complete curriculum. Instead these materials are designed to provide background for the practicing teacher in the use of appropriate concrete models. We hope that completing these activities will lead to a deeper understanding of computation process for both the teachers themselves and the students in their classes.

The first section describes various types of base-10 materials which can be used in completing the activities. In a classroom situation, each student should have their own supply of base-10 materials.

To effectively use the activities described in a classroom setting, the teacher must try out each activity ahead

of time to be sure they understand the manipulations involved. Additional examples should be prepared as well, beginning with easier problems and working towards more complex examples.

Each activity is organized to begin with concrete experiences using physical materials moving towards more abstract examples using pictures and number symbols. Older students may require only a brief review of the operation (or perhaps a demonstration) before moving on to symbolic problems. It should be remembered however that students learn most from experiences actually manipulating the concrete models. Each student should work towards a personal understanding of the concepts and techniques associated with computation.

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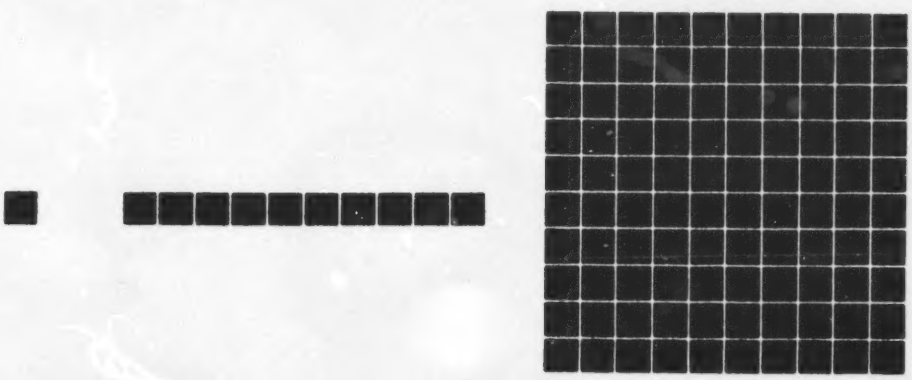
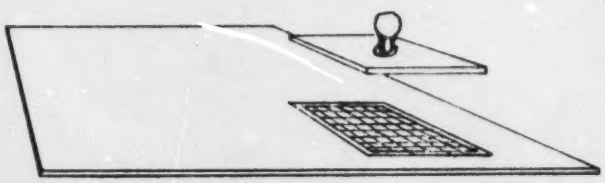
ALTERNATIVE BASE-10 MATERIALS

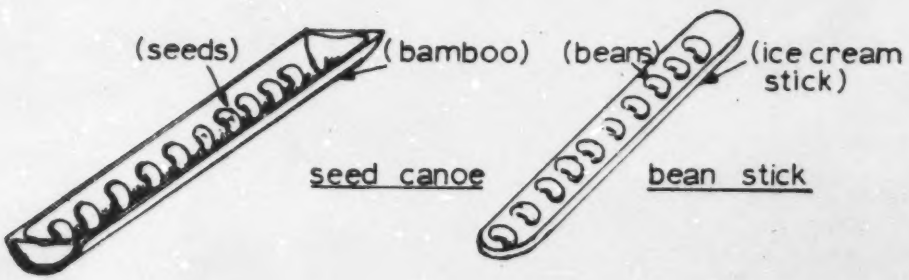
The materials used throughout is series of computation activities consist of base-10 blocks and organizer mats. Base-10 models can be constructed out of many materials. Commercial blocks based on a 1 centimeter unit are made of wood or plastic.



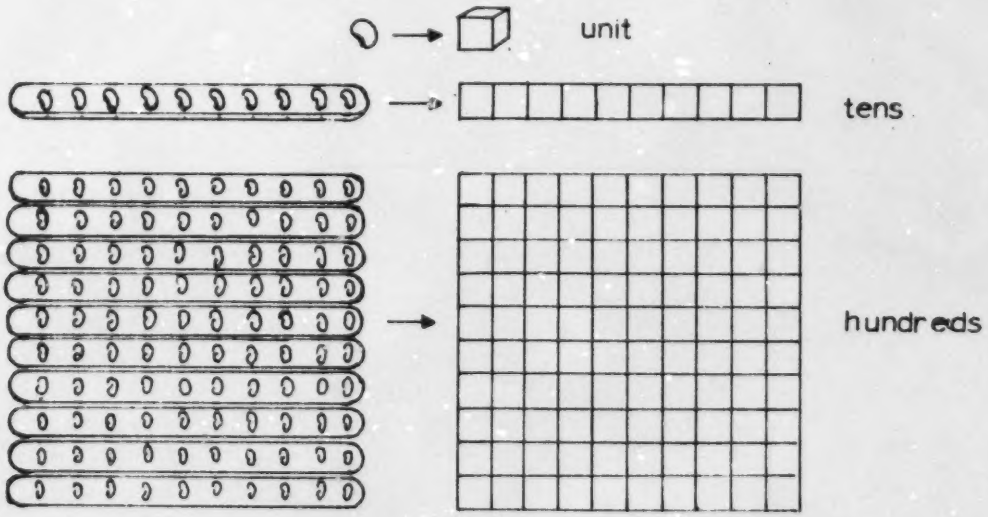
Home made materials can easily be constructed with a set of stamps (unit, 10-rod, 100 raft), card board cartons and a sharp knife.



Other possibilities include gluing exactly 10 seeds or dried beans on flat pieces of bamboo or icecream sticks to make "seed canoes" or "bean sticks."

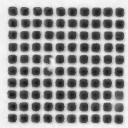


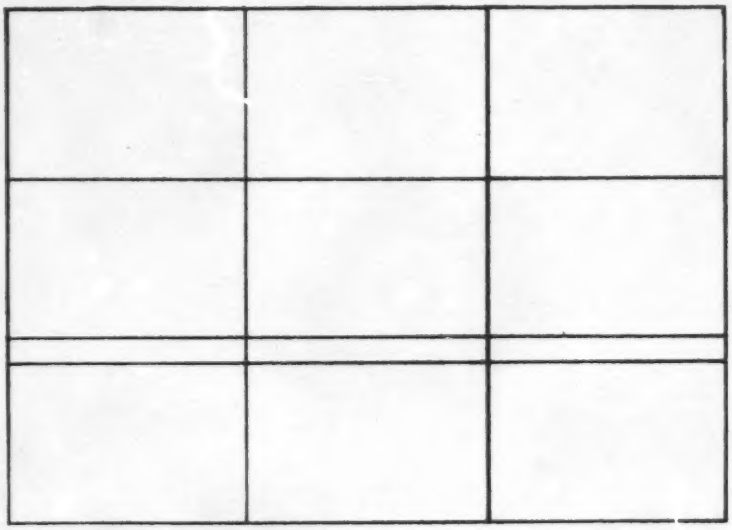
Putting ten canoes (or bean-sticks) together into a raft by gluing two sticks along the back gives a model for the 100's place. Separate seeds or beans are used as the unit.



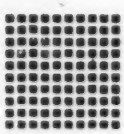
In addition to the base-10 materials the place value mats pictured below are useful in developing computation algorithms.

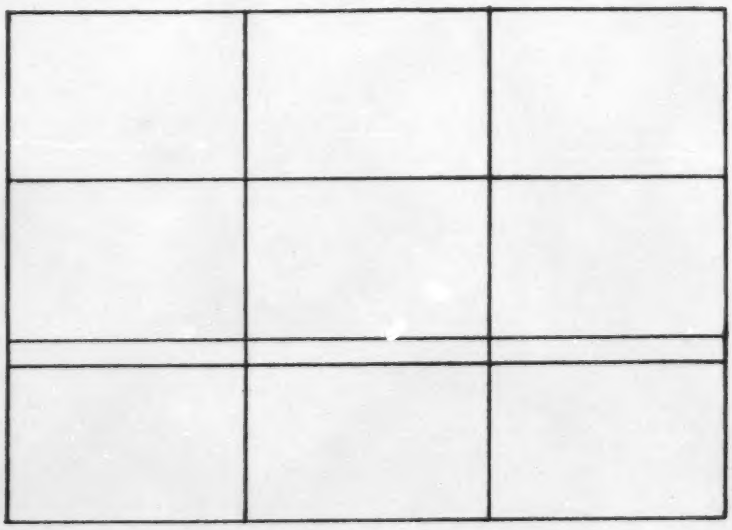
Addition Mat (see page 37)



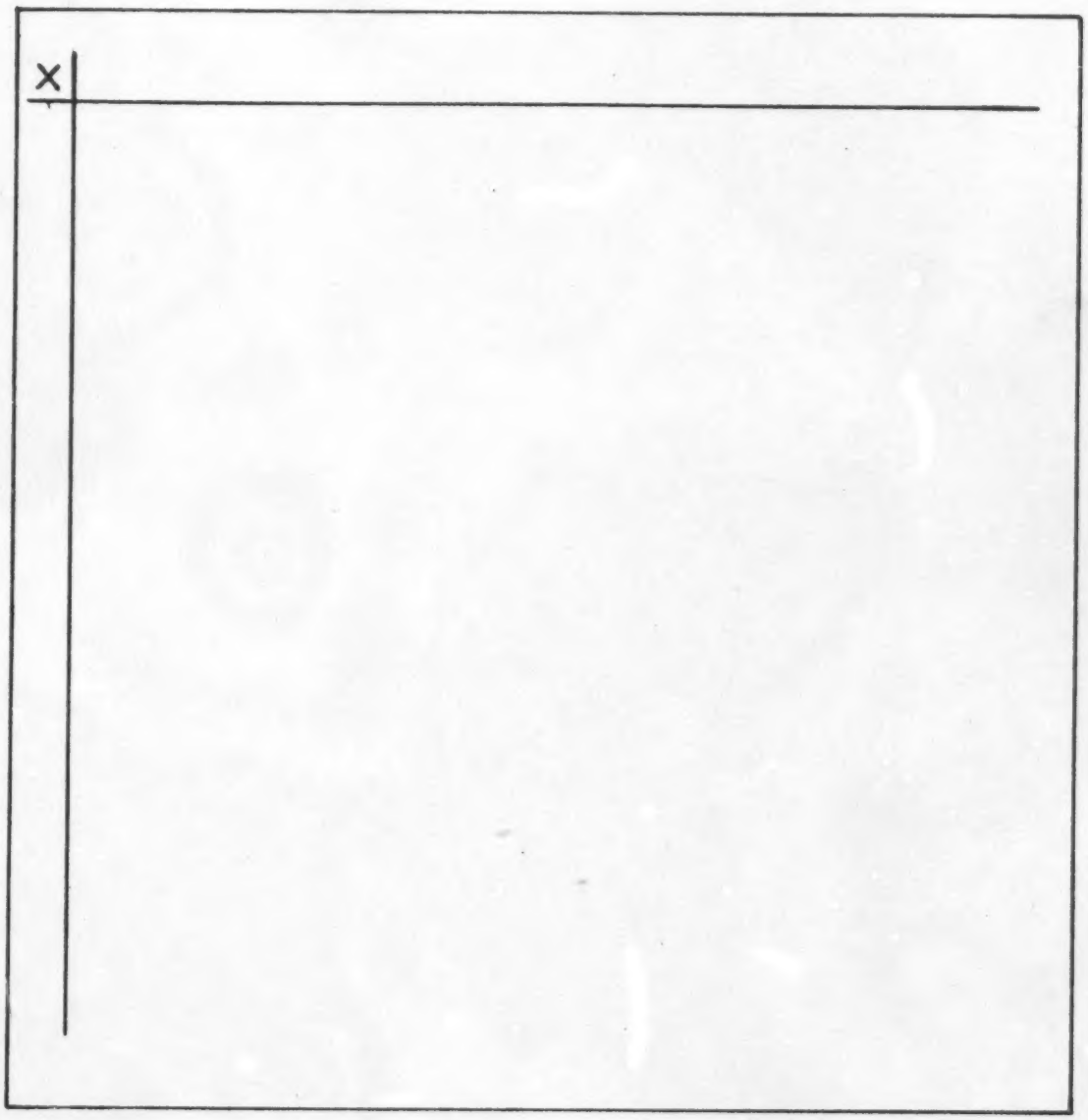


Subtraction Mat (See page 40)





Multiplication and Division Mat (See pages 42 & 46)



Place-Value

Activity 1.

Procedure:

Give each student 5 rods and 15 units. Ask the group to show a specific number - eg. 12 - as quickly as possible. Some students may rapidly count out twelve individual units while others may use a rod and units. Repeat this activity naming larger and larger two-digit numbers, hence making it increasingly tedious to count out the entire number with single units. At some point during this process, students should begin using a 10-rod instead of ten individual blocks. Once this happens, stop and talk about the "best" or "quickest" way to pick up ten units. Ask which is quicker, picking up ten single blocks or one 10 rod. Students will soon understand the efficiency of selecting a 10-rod instead of ten unit blocks. Now ask the whole group to pick up eighteen blocks the "best way". Continue to ask students to show sets of blocks under twenty until they are comfortably and consistently picking up a 10-rod to represent ten.

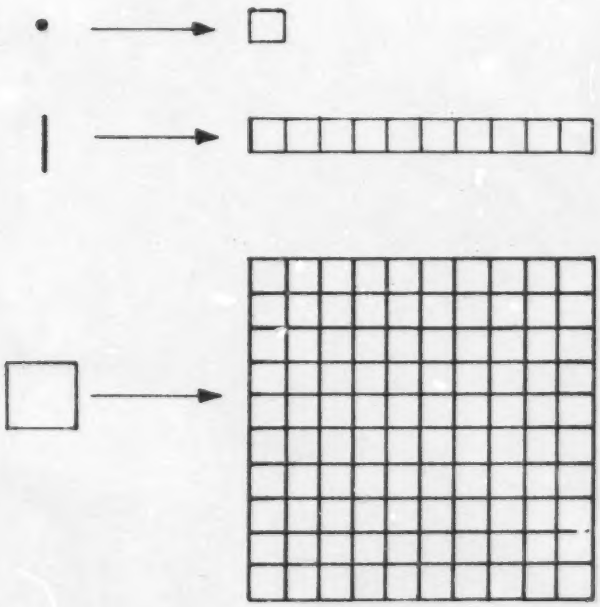
When asked the "best way" to show 24 (or any two-digit number greater than 20), students may tend to select only one rod and count the remainder in single units. If they don't progress on their own to picking up a 10-rod for every group of ten, the teacher should talk about that possibility and arrange an example, asking, "Is mine the same number of tiles as yours? If so, which way is quicker?"

Student's Teacher's

Continue this activity until students are consistently picking up 10-rods for every group of ten.

Activity 2.

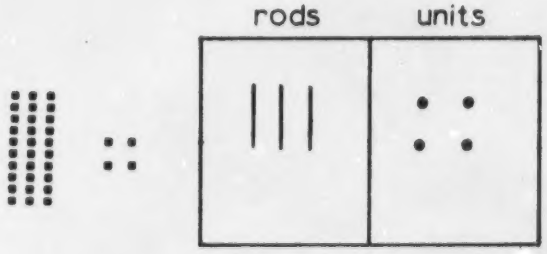
Note: For this activity it is necessary to develop a short-hand notation to represent the base-10 blocks. The following system works well.



Procedure:

Give each student access to a pile of unit blocks and 10-rods. Ask questions similar to those in Activity 1 above. Let students review the "best way" of picking up numbers of blocks until they seem confortable selecting a 10-rod for all groups of ten.

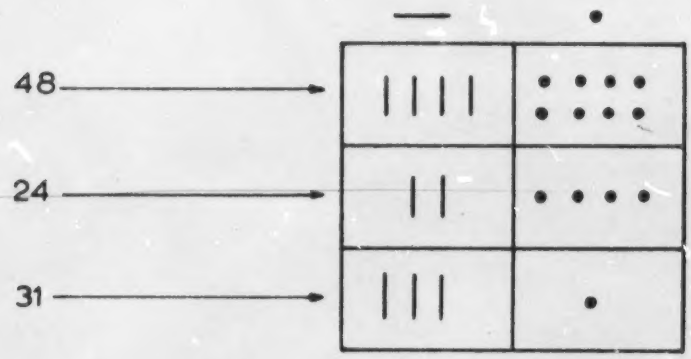
Next, ask the children to show you a specific number (32 perhaps). After they have each found the "best way" (i.e., three 10-rods and two unit blocks) ask them to record the results using the shorthand notation as below:



Continue this procedure with increasingly larger numbers. As they become familiar with the notation, students may want to use shorthand notation directly, without using the blocks. This progression should be encouraged. Remove the base-10 blocks when students no longer need them as a model, and continue to pose "best way" problems, asking students to use only the shorthand notation.

Complete Activity Sheet 2 (page 35)

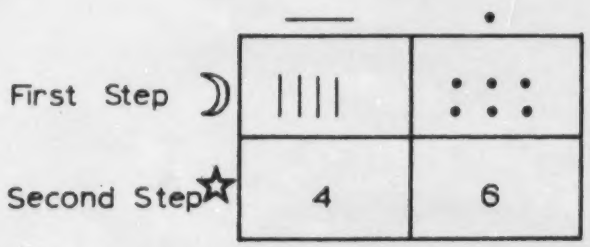
Teacher says Students write:



Activity 3.

Procedure:

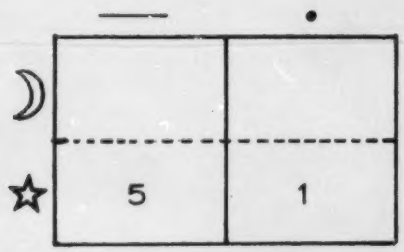
Have the students complete Activity Sheet 3 (page 36). Call out numbers, asking students to record it in the "best way" (short hand notation). These notations should be made in the row following the moon. Next, ask students to record the number of 10-rods and Units represented by each result. The notation should be made in the row following the star as below:



forty six

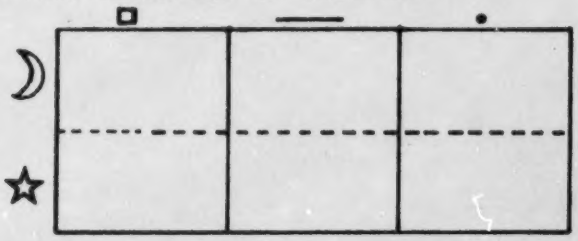
Continue using many different numbers, always asking students to verbalize the number that appears in the star-row.

Now prepare the same type of record sheet, but this time call the numbers orally (ie. "fifty one"). Have the student first write in the symbol in the star-row as below then fill in the shorthand (lines and dots) in the moon-row.



fifty one

The preceding activities may easily be extended to include the hundreds place. As mentioned previously this piece is referred to as a "100-raft". The teacher who wishes to include hundreds in activities 1, 2 and 3 may call out numbers greater than 99 after students have proved themselves capable of consistently picking up 10-rods for sets of ten, twenty, thirty, etc. The record sheets maybe modified to include hundreds as follows:



Note: Similar activities can be developed using money. Ask the students tc figure out the "best way" to carry around different amounts of money using only 1 toes, 10 toea and 1 kina coins. The "best way" means the fewest number of coins needed for a given amount of money.

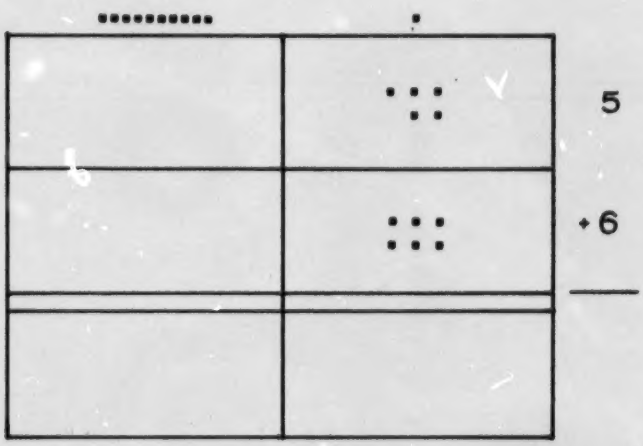
Addition

Activity 4.

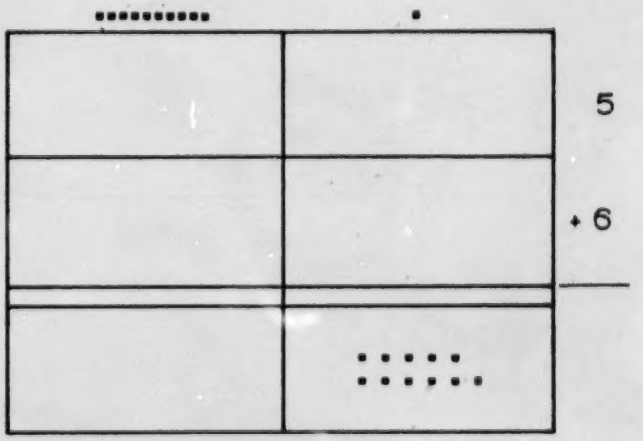
Procedure:

To help develop an understanding of the regrouping process each student should have a set of base-10 materials and an Addition Mat (see page 37).

Begin by having each student set up the problem: $\begin{array}{r} 5 \\ +\ 6 \\ \hline \end{array}$

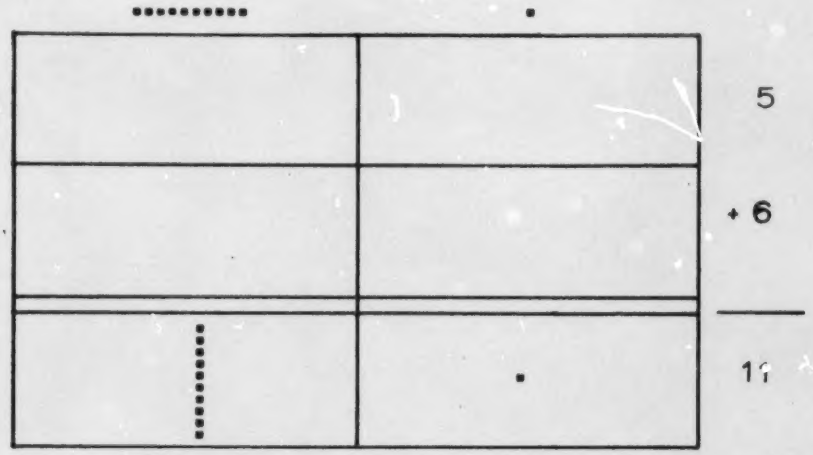


Next have them slide both sets to the bottom row. Point out that the pieces must be placed in only their correct column (units in the ones column, 10-rods in the tens column, etc).



Now ask if there is a way to have the same number of cubes (eleven) but with fewer pieces. If the students understand the place-value activities they should realize that by trading.

in 10 units for one 10-rod (along with the one left over) will have the same amount as the eleven original units. By making the trade however, they will have fewer pieces to worry about.



This idea of having the fewest number of pieces to carry around (in your pocket or in your mind) is a good way to help students understand the basic rules for place value and arithmetic.

Activity 5. Two-Place Addition with Regrouping.

In most mathematics texts we generally find two-place addition without regrouping introduced before and apart from problems that require regrouping. However since these two types of problems involve only one process children do not view them differently. Adding twenty-two and thirteen pieces of candy is not fundimentally different from adding twenty-two and nineteen pieces. Furthermore, when addition without regrouping is introduced separately, children are not required to recognize the place-value relationship of two-digit numbers. For example the problem

$$\begin{array}{r} 28 \\ +\ 41 \\ \hline \end{array}$$

becomes 8 + 1 and 2 + 4.

Because the concept of place value is lost when the two digits are thought of as separate entities, children understandably encounter difficulty when they must eventually learn to regroup.

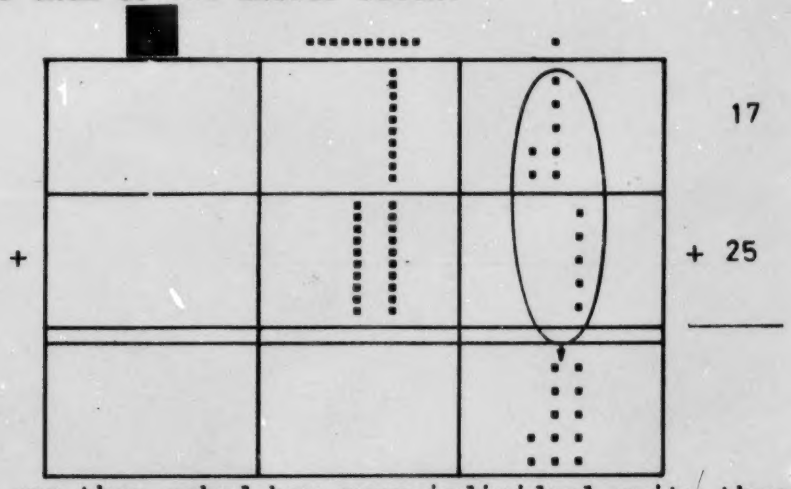
Suddenly, the two digits must be related in a very specific way and cannot be dealt with separately.

To avoid this confusion, two-digit addition with regrouping and non-regrouping are viewed as a natural and intergral part of the process of addition.

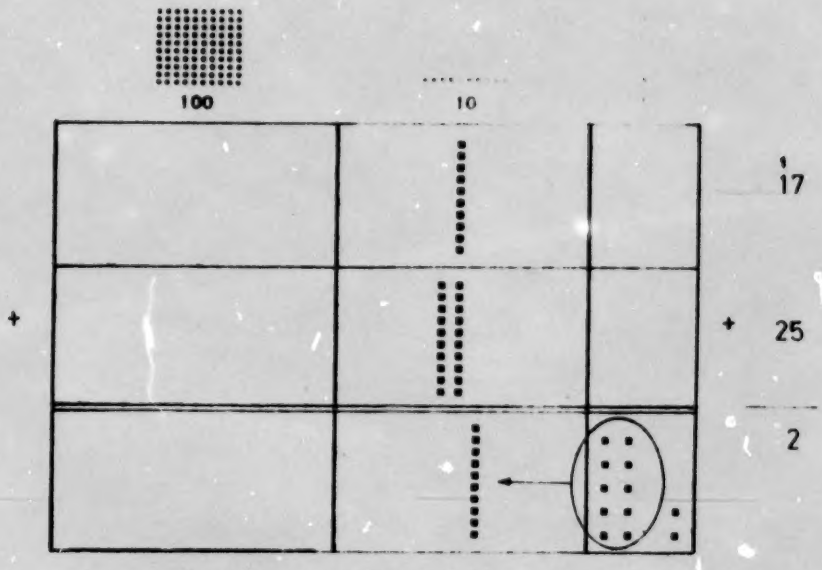
Procedure:

Students are given base-10 materials and Addition Mats (page 37) and asked to work problems that are given to them verbally. Again, place values are represented on the mat by columns for single units, 10-rods and rafts of one hundred.

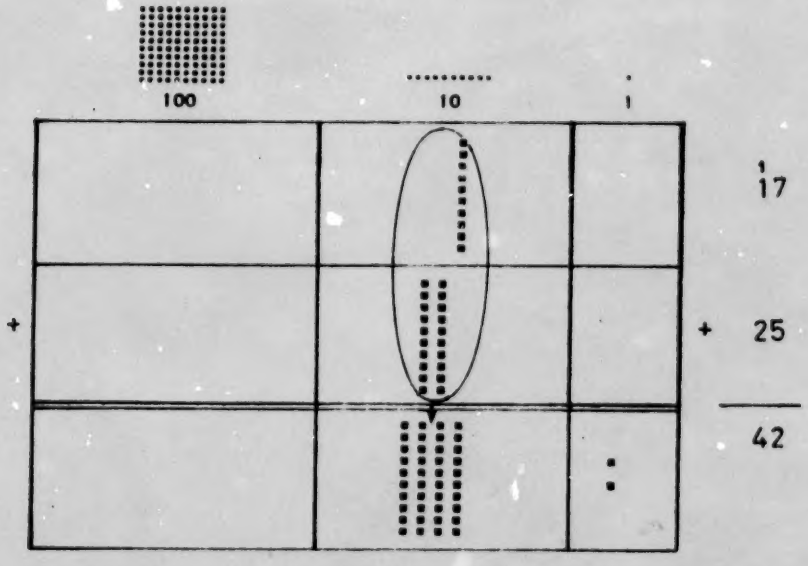
When asked, for example, to add seventeen and twenty-five, students first combine all the single units (ones) and move them to the answer block.



Student are then asked how many individual units there are in all and whether this is the "best way" to show twelve. They should respond that the best way would be by means of a 10-rod and two units.

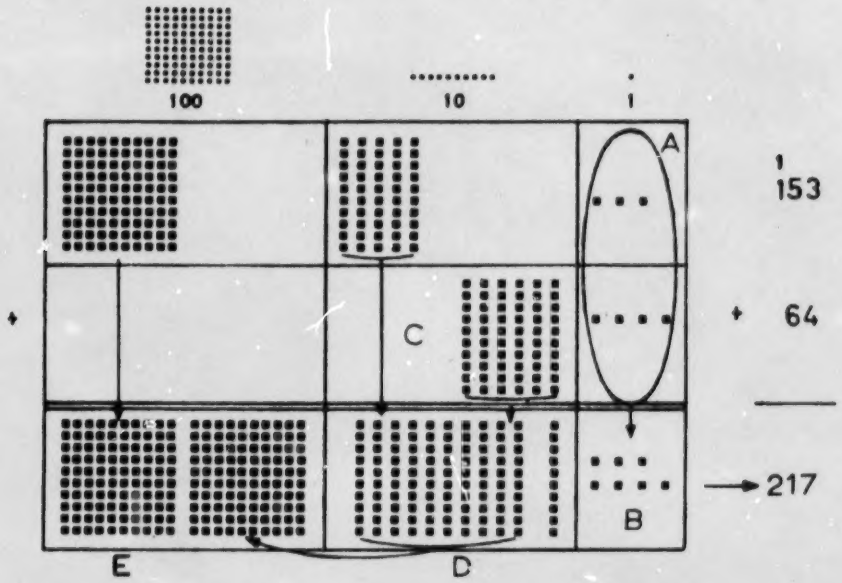


The same process is then repeated for the ten's column.



The resulting group of 10-rods (including the one regrouped from the units place) is examined to see whether it, too, is the "best way" to show forty. Since in this case forty is already represented correctly, the problem is completed. The answer is forty-two.

Here is how the sum 153 + 64 would be handled:

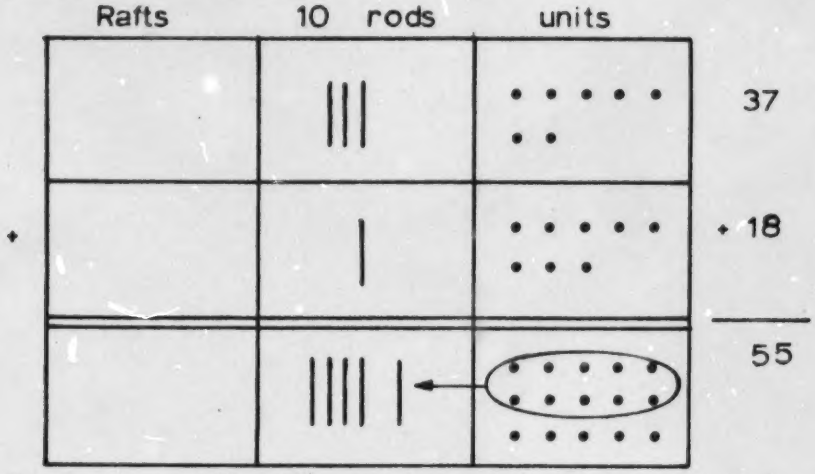


All the units are combined (A) and then checked to see whether the sum is represented in the "best way" (B). In this instance it is represented in the best way, so we leave it alone. Next all the 10-rods are combined (C). This time we must change the 10-rod group to the "best way" (D) by trading ten for a raft of one-hundred. The raft is placed in its column (E). The other raft is brought down to the answer block where it is again checked for the "best way". The answer is 217.

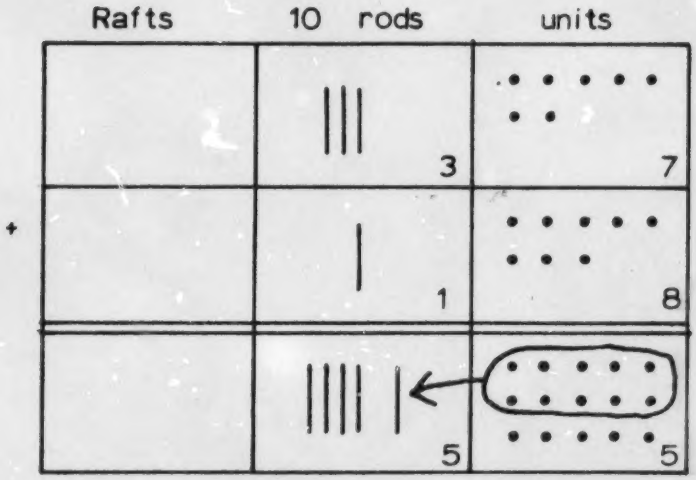
ACTIVITY 6.

Using Activity Sheet 6A (page 38), have students complete 2 and 3 digit addition problems. Give the problems orally or have the students make up their own problems. The problems should be completed using shorthand notation (dots, lines and squares).

The example below shows 37 + 18:



Students can write the number symbols next to the grid as above or could write the symbols inside the grid squares.



Finally have the students complete Activity Sheet 6B (page 39). Students should practice problems of this type until the place-value grids are no longer needed. The amount of drill will vary from student to student.

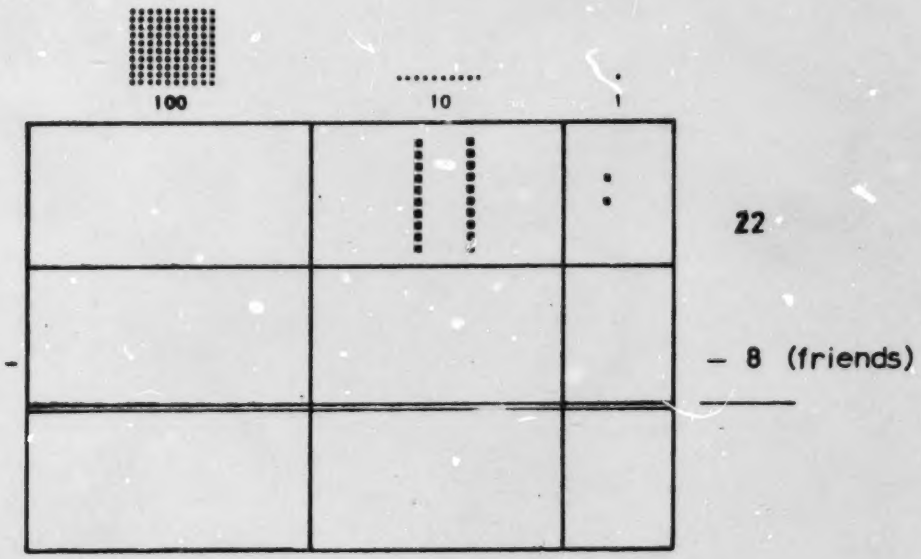
Subtraction

ACTIVITY 7.

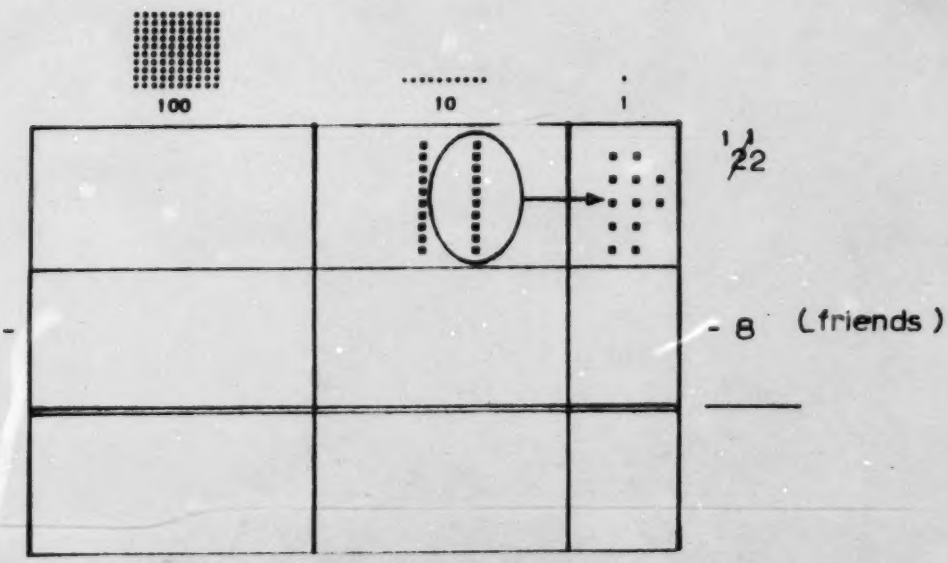
Procedure:

Using a Subtraction Mat (page 40) have the students set up the problem:

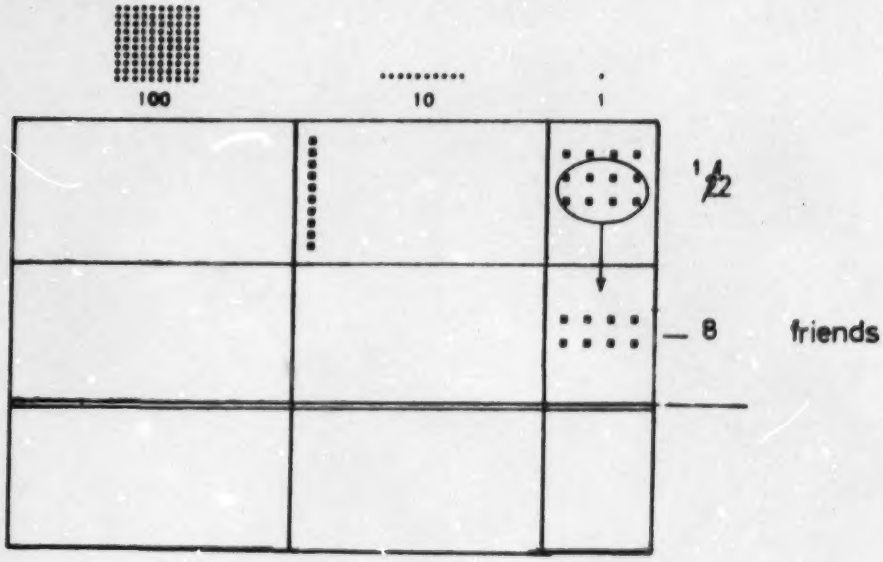
$$\begin{array}{r} 22 \\ -\ 8 \\ \hline \end{array}$$



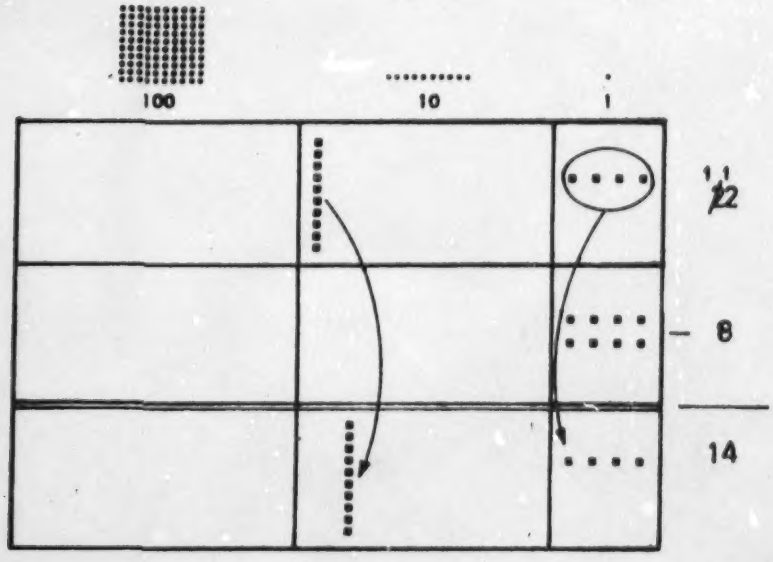
Each "friend" is given one unit. Since there aren't enough single units for all eight friends we must breakup (regroup) one 10-rod into the one's column.



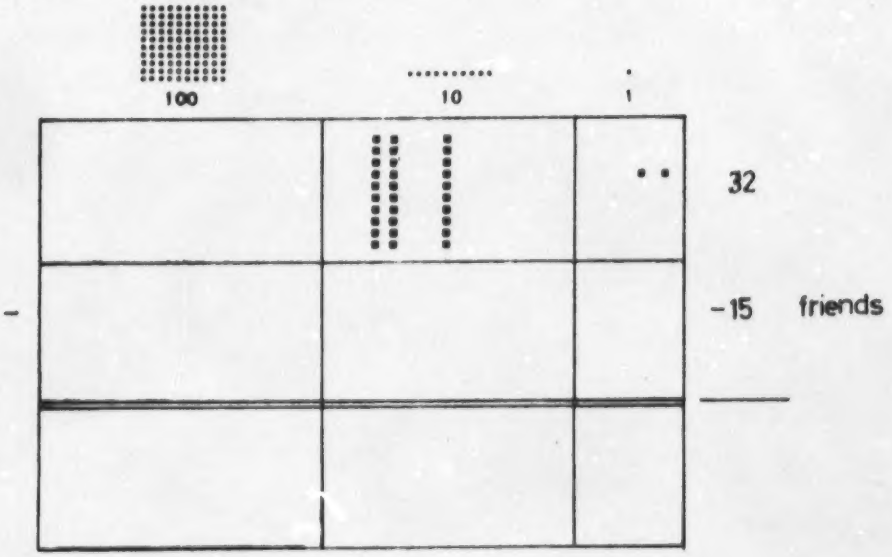
Now each friend can be given one:



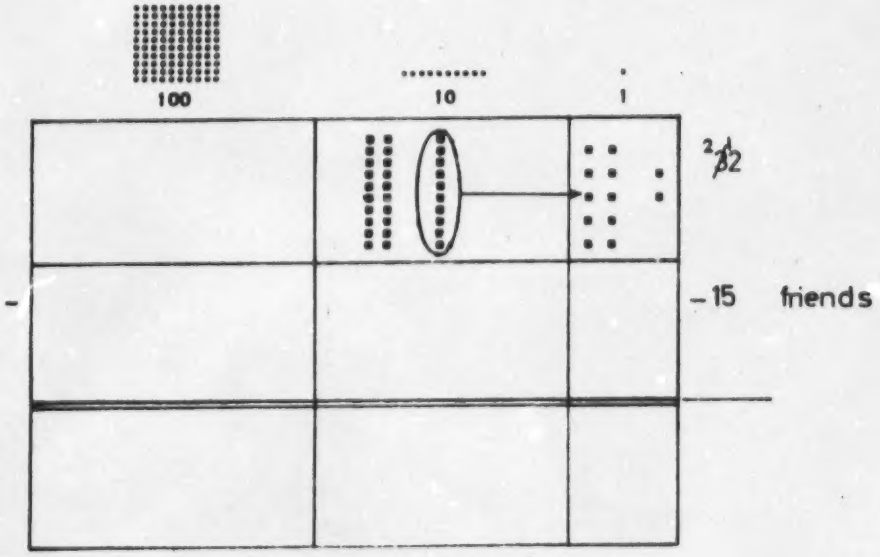
The remaining blocks are moved to the bottom row giving the answer 14.

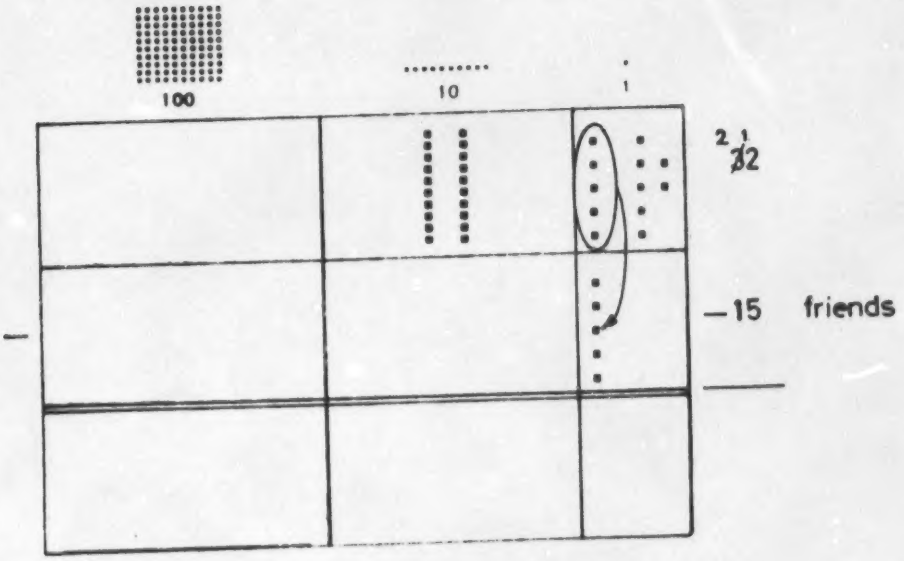


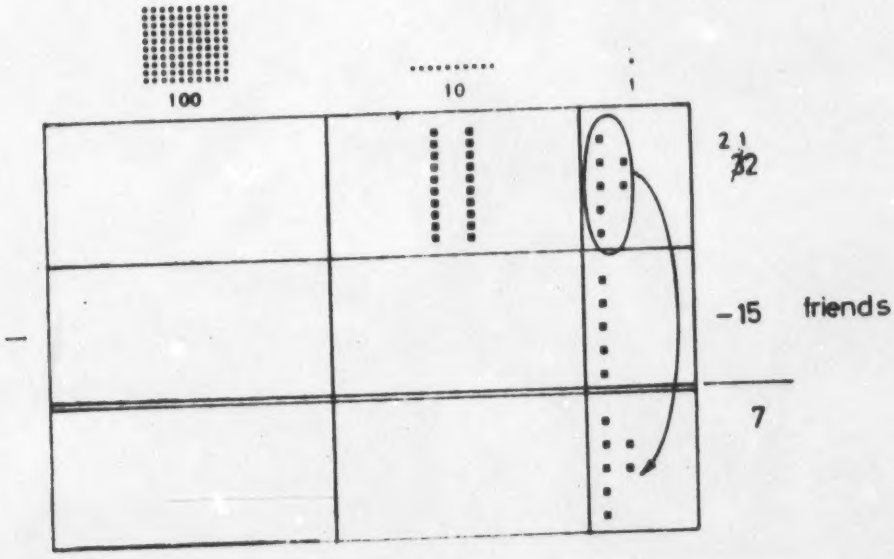
Here is another example.



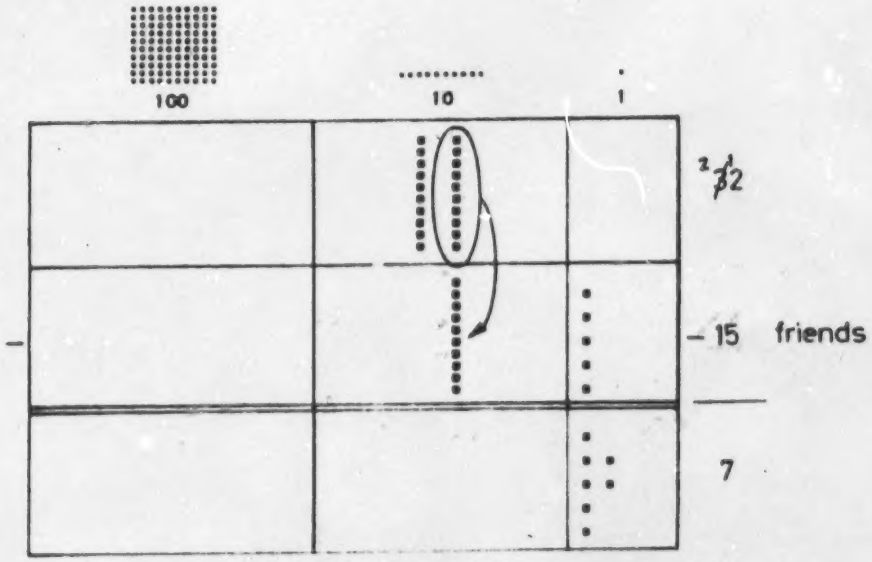
First we regroup to give five friends one unit each.



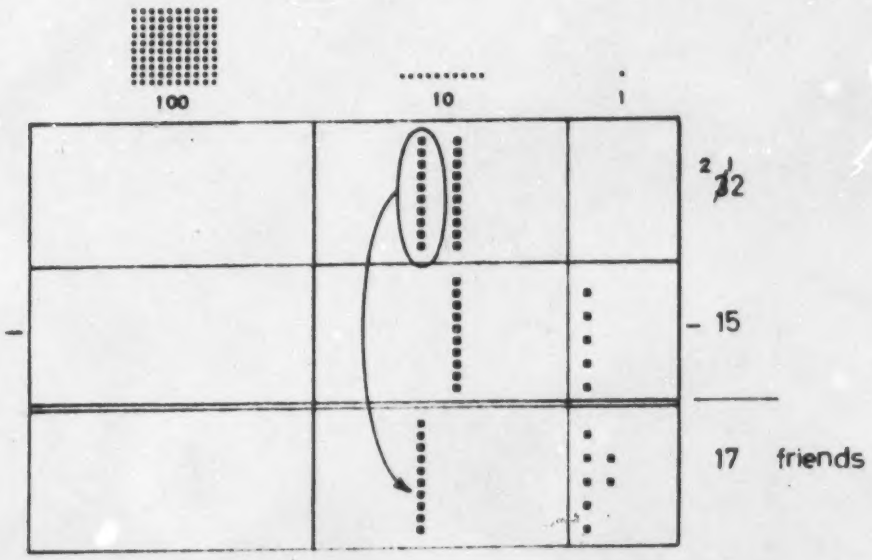

100
10
1
21
32
– 15
friends


100
10
1
21
32
– 15
friends
7

Then we give the remaining ten friends a whole 10-rod so they each get their share.



The remaining 10-rods brought to the bottom giving the answer 17.

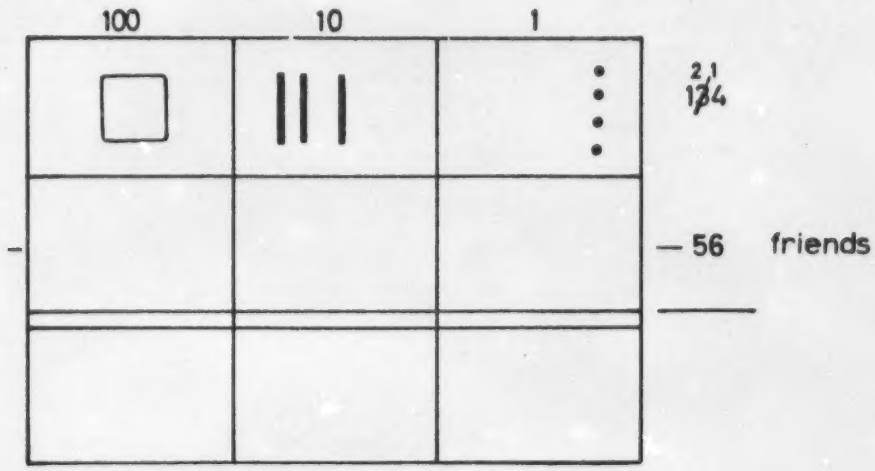


ACTIVITY 8.

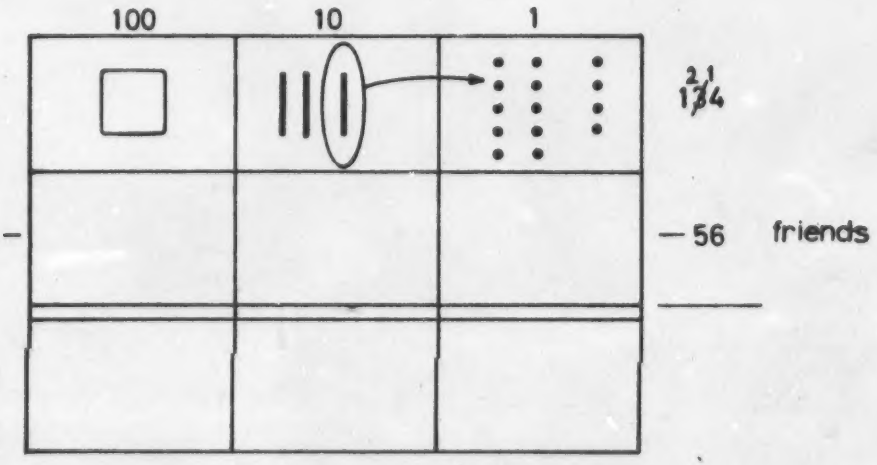
Procedure:

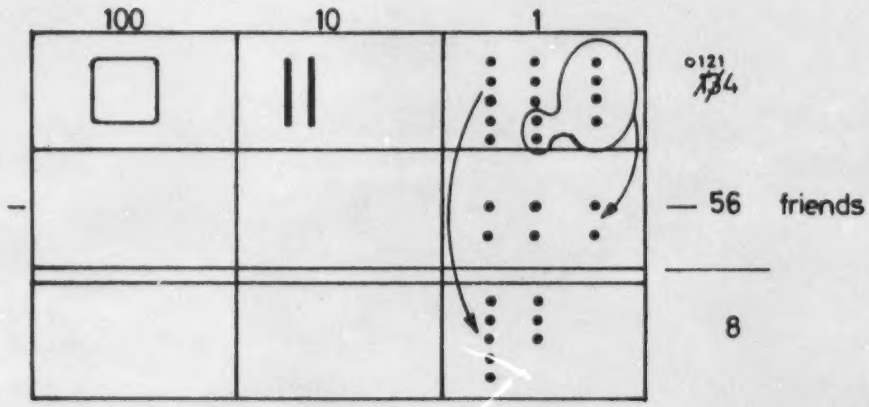
Carry out similar subtraction problems using shorthand symbols (see Activity sheet 8 page 41).

Example:

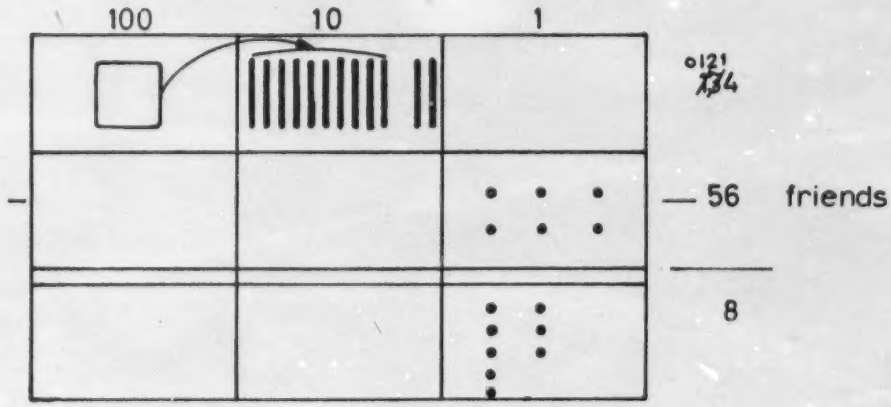


First regroup to give the 6 friends one unit each and move remainder to bottom.

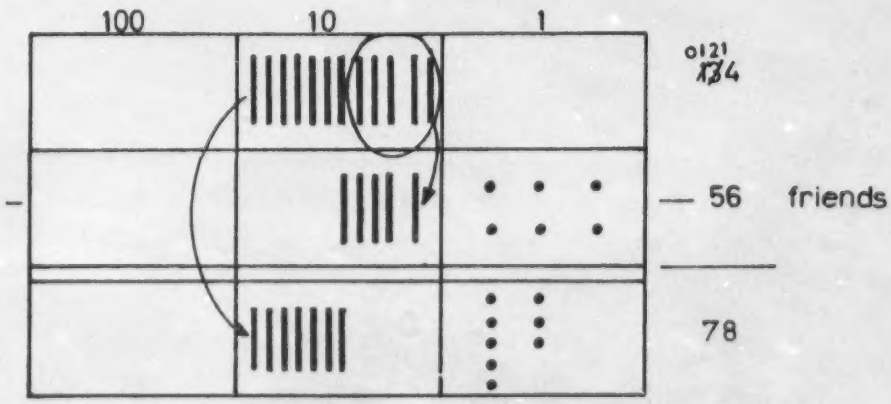




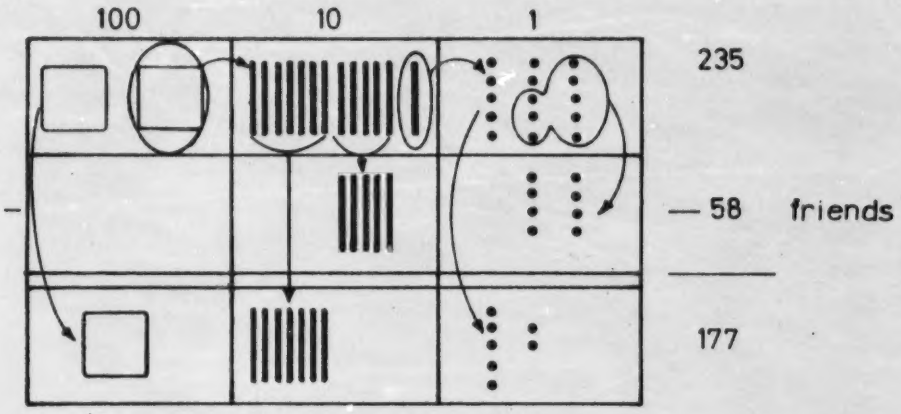
Since there are only two 10's left and you need five 10's to share with the 50 remaining friends, regroup the 100-raft.



Now share five 10's with the 50 remaining friends and move the rest to the bottom.



With practice the whole problem can be done on one mat.



Using the base-10 blocks and shorthand symbols helps students understand the process of regrouping for both addition and subtraction. With practice students will find it easier to use the standard base-10 numerals to complete their problems. When students get a wrong answer to a problem encourage them to use the techniques above to correct their mistakes.

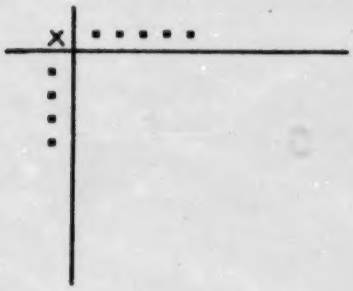
MULTIPLICATION

ACTIVITY 9.

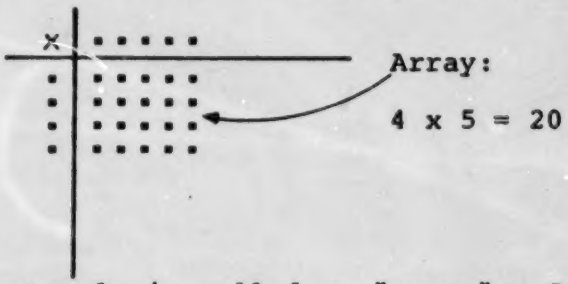
Procedure.

Give each student a set of base-10 materials and a Multiplication Mat (page 42). Have students first set up simple problems like:

$4 \times 5 = ?$

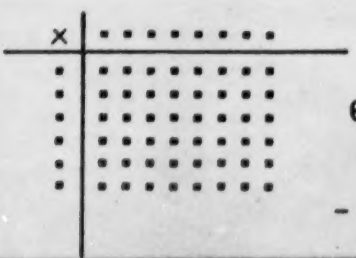


To find the answer have each student build a rectangle of units with four rows and five in each row.



This rectangle is called an "array". Counting the number of blocks in the array (20) gives the answer for 4 x 5. Have your class build all the arrays 1 x 1 through 9 x 9.

Using shorthand notation makes it easy to keep a record of arrays. Give each student Worksheet 9B (page 43) and have them complete the times table.



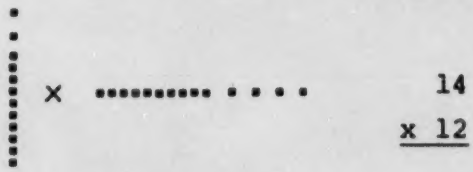
$6 \times 8 = 48$

Multiplication of a Two-Digit by a Two-Digit Number

ACTIVITY 10.

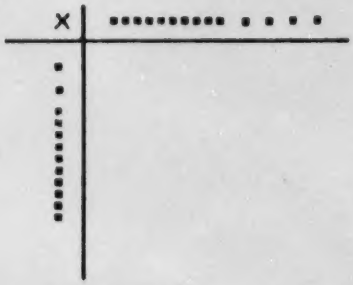
Procedure:

Have students use base-10 materials to solve the following problem:

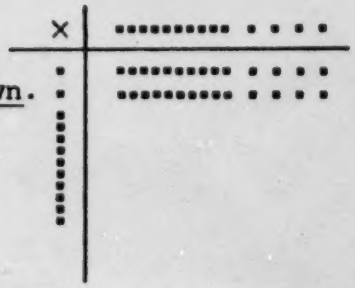


14
x 12

Set up the problem as follows:

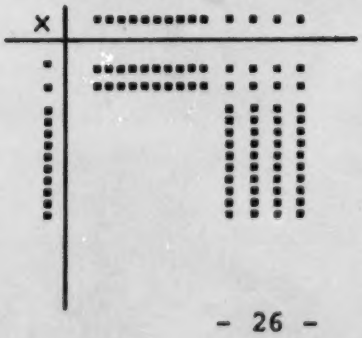


The next step is to fill in the array with units, 10-rods and 100 rafts start with the top row first.

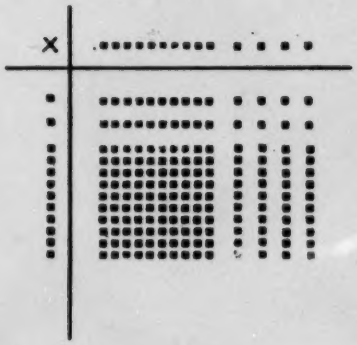


Students fill in array as shown.

Students should note that 10-rods should be used because they are more convenient than sets of ten individual units.



Finally, students find that the remaining space can be filled by a raft.

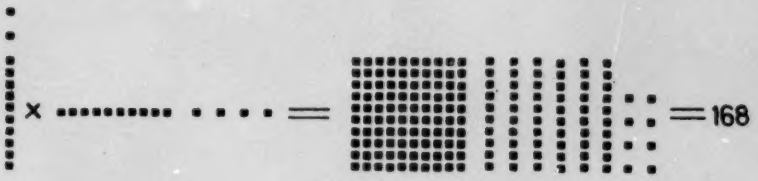


14
12

8
20
40
100

168

Note that this procedure produces the correct number of equal rows to make the array. Therefore the answer is:



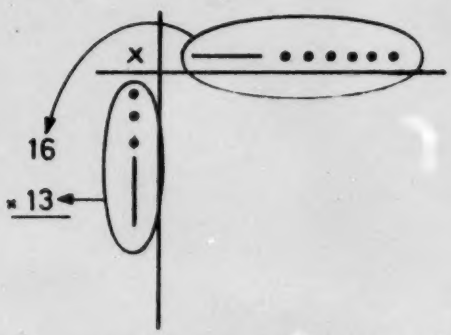
ACTIVITY 11.

Procedure:

Here is an example using shorthand symbols.

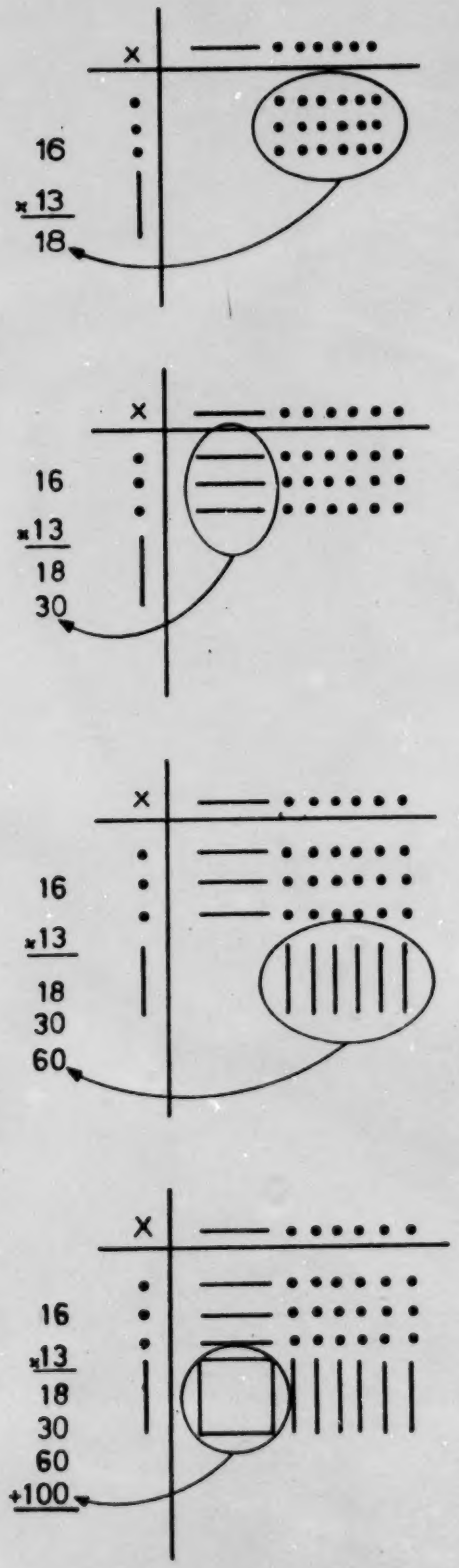
13 x 16 = ?

Step 1. The set up:



Step 2. Filling in:

Using the same procedure as in Activity 10, students fill in the array starting with units and moving to rafts. Students should record each partial product as they fill it in.

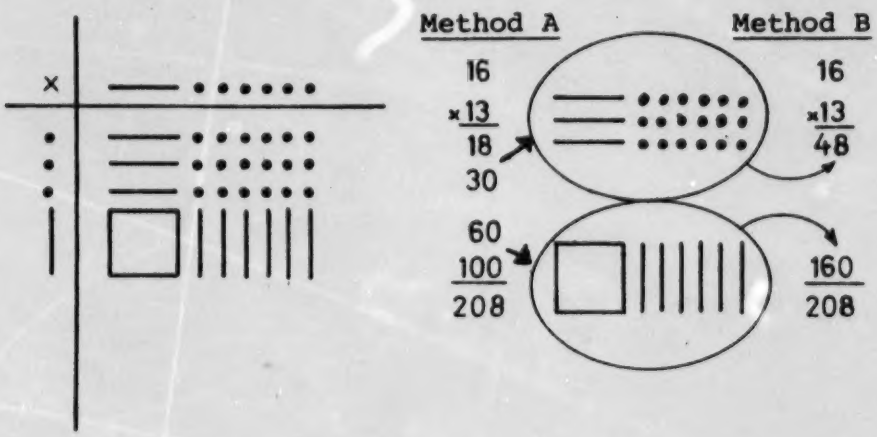

x
16
x13
18
x
16
x13
18
30
x
16
x13
18
30
60
x
16
x13
18
30
60
+100

Step 3. Solution:

Adding the partial products gives the final answer.

$$\begin{array}{r} 16 \\ \times\ 13 \\ \hline 18 \\ 30 \\ 60 \\ +\ 100 \\ \hline 208 \end{array}$$

Combining the first two and last two partial products as below gives the standard multiplication algorithm.



Note that Methods A and B above give the same answer.

After experience with several worksheets like Activity Sheet 11 A and 11 B (pages 44 & 45), students should no longer need to rely on the shorthand representation of each problem. Some students will require more practice then others before they will be able to multiply using only the number symbols. Here is a review of the steps involved:

A. Beginning with the units-place multiplier, multiply 3 x5:

$$\begin{array}{r} 45 \\ \times\ 23 \\ \hline 15 \end{array}$$

B. Next 3 x 40:

$$\begin{array}{r} 45 \\ \times\ 23 \\ \hline 15 \\ 120 \end{array}$$

C. Since we have completed multiplying by the units place, we now multiply 20 x 5:

$$\begin{array}{r} 45 \\ \times\ 23 \\ \hline 15 \\ 120 \\ 100 \end{array}$$

D. Finally, we multiply the ten's place times then ten's place- 20 x 40:

$$\begin{array}{r} 45 \\ \times\ 23 \\ \hline 15 \\ 120 \\ 100 \\ 800 \end{array}$$

E. We then add the partial products to find the answer:

$$\begin{array}{r} 45 \\ \times\ 23 \\ \hline 15 \\ 120 \\ 100 \\ +\ 800 \\ \hline 1035 \end{array}$$

This method of multiplication (Method A) is somewhat easier to learn than the standard method (B) and seems just as useful.

Division

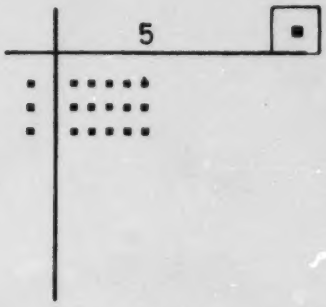
ACTIVITY 12

Procedure:

Give each student base-10 materials and a Division Mat (page 46). To solve problem $3\overline{)16}$ we make an array with three equal rows using as many of the units as possible.

Start with: ..... 16 units
.....
.....

Make an array with 3 equal rows.

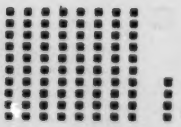


$$3\overline{)16} \quad 5 \text{ R}1$$

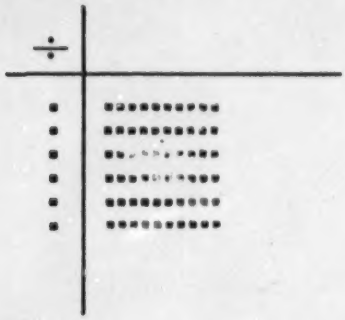
The answer is 5 remainder 1. Note that the division process is the opposite of multiplication. Here is an example using larger numbers and base-10 materials:

$$6\overline{)84}$$

Start with 84:

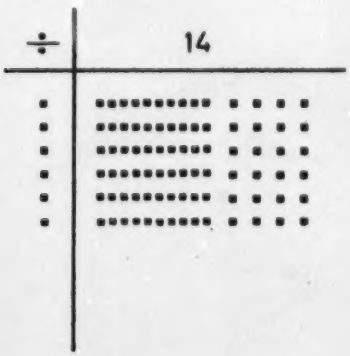


Set up the problem on the division mat. Fill in the six row array using the largest pieces first. Remember to keep the rows equal.



$$6\overline{)84}\quad 10,\ 60,\ 24$$

This first step used up 60 of the original 84 with 10 in each row and 24 left. We now distribute the remaining 24 evenly amoung the six rows. It is necessary to trade the two 10-rods for units to keep the rows equal.



$$6\overline{)84}\quad \boxed{14},\ 4,\ 10,\ 60,\ 24,\ 24,\ 0$$

(add up)

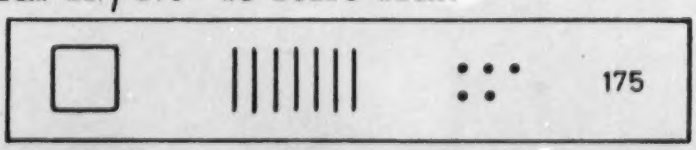
Since all 84 are used up the answer is 14.

ACTIVITY 13.

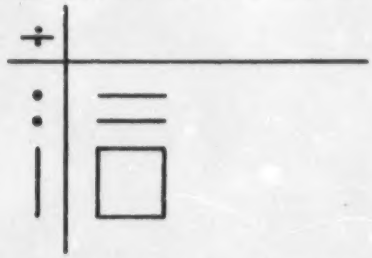
Procedure:

Have the students use shorthand notation for larger problems. Have them complete Activity Sheet 13 (page 47).

For the problem $12\overline{)175}$ we start with:



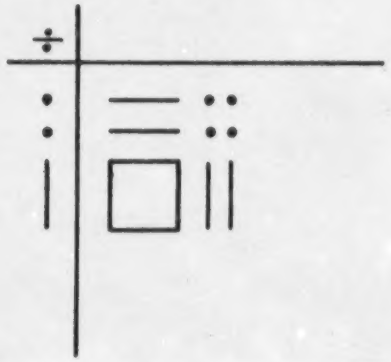
Set up the problem on the division mat.



```
      10
  12 ) 175
       120
       ---
        55
```

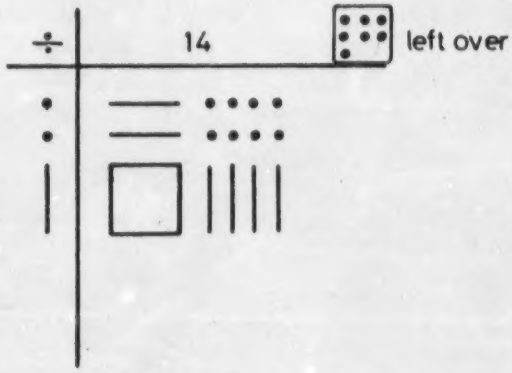
First we fill in the array with the largest pieces possible (one 100-raft and two 10-rods) giving 10 in each row. There are 55 left to distribute.

We could trade all the 10-rods for units but notice a quicker arrangement below.



```
       2
      10
  12 ) 175
       120
       ---
        55
        24
       ---
        31
```

We now have an array with 12 in each row and 31 left to distribute. Trading in one 10-rod allows us to continue:




```
      14
       2
       2
      10   (add up)
  12 ) 175
       120
       ---
        55
        24
       ---
        31
        24
       ---
         7
```

With practice estimating the partial quotients, the method above is identical to the standard algorithm.

Examples:

1.)

```
   [17]
     7
    10   ↑ add up
16 ) 272
    160
    112
    112
      0
```

2.)

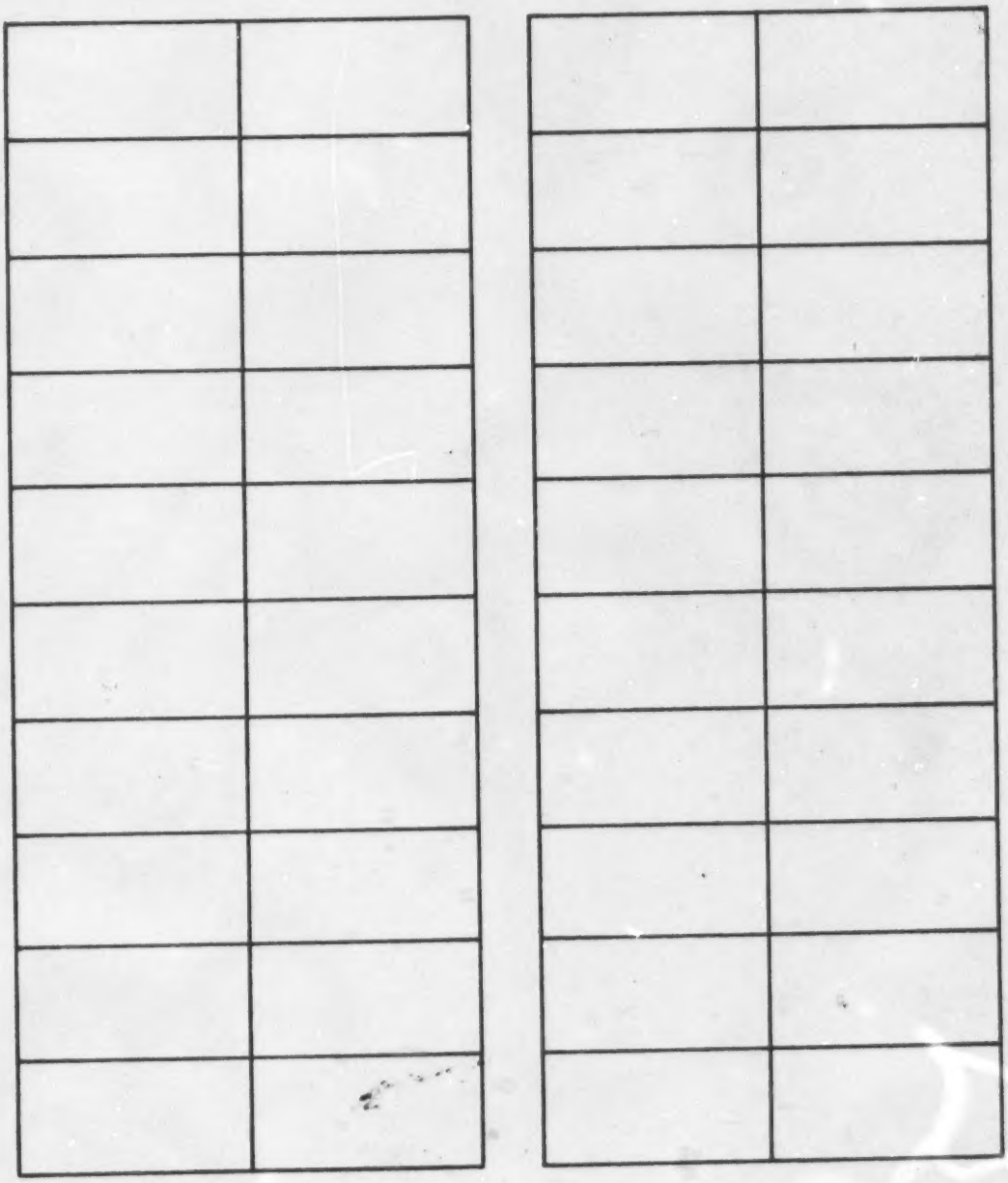
```
   [24]
     4
    20   ↑ add up
24 ) 576
    480
     96
     96
      0
```

Division by more than two digit divisors should not be attempted using base-10 materials or shorthand symbols since the process becomes quite complicated. It is better to practice with examples like those above and once the process of division is understood, larger problems can be undertaken using standard symbolic methods.

Place-Value

Activity Sheet 2.

Name: _____



Place-Value

Activity Sheet 3.

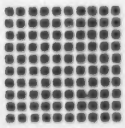
Name: _____

	▬	●
☆		
☆		
☆		
☆		
☆		

ADDITION MAT

Activity Sheet 5.

Name: _____



100

10

1

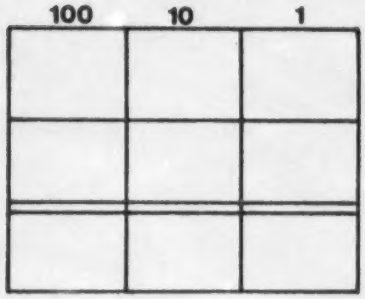
+		

Addition

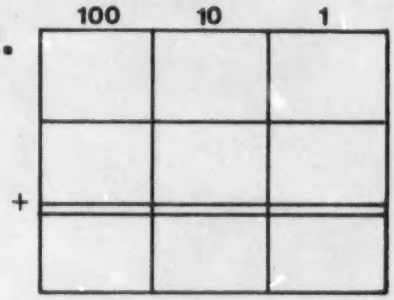
Activity Sheet 6A.

Name: _____

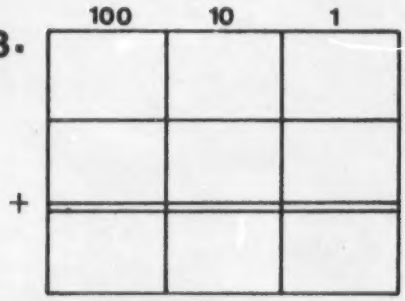
1.



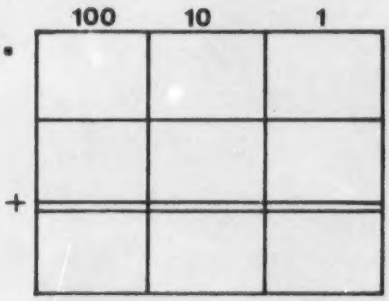
2.



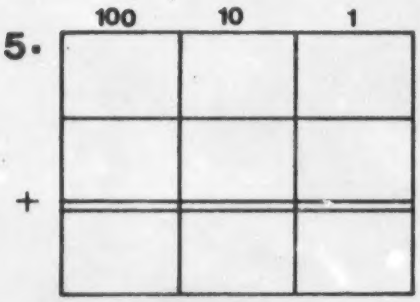
3.



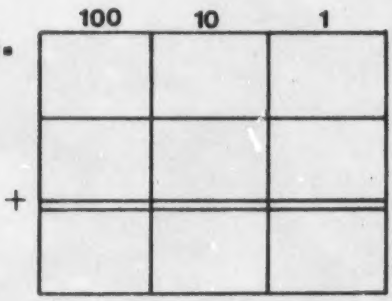
4.



5.



6.



ADDITION

Activity Sheet 6B

Name: _____

Example:

100	10	1
	3	4
	2	8
		6

34
28
6

1.

100	10	1
1	2	6
	9	2

126
92

2.

100	10	1
1	2	7
2	1	1

127
+211

3.

100	10	1
5	2	9
	7	8

529
78

4.

100	10	1
2	9	0
3	7	8

290
+ 378

5.

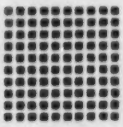
100	10	1
7	0	8
	9	8

708
98

SUBTRACTION MAT

Activity Sheet 7.

Name: _____



100

10

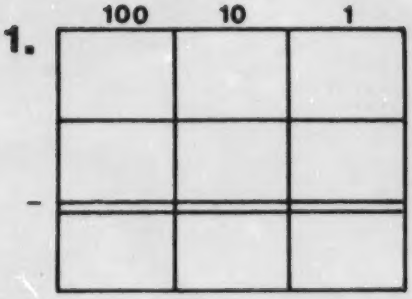
1

SUBTRACTION

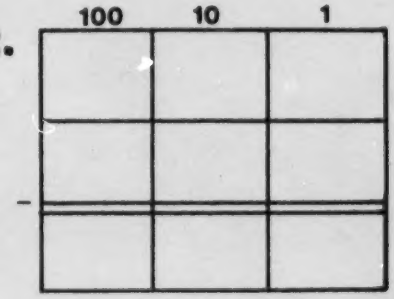
Activity Sheet 8.

Name: _____

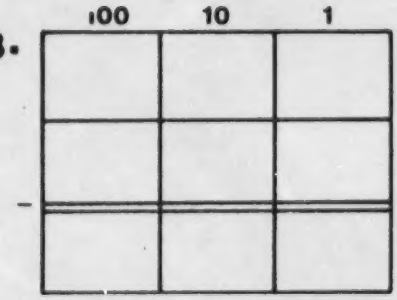
1.



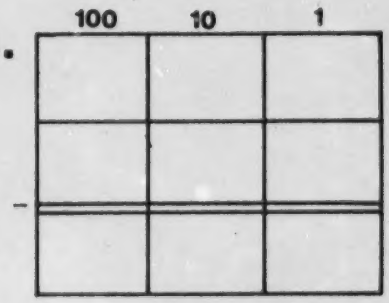
2.



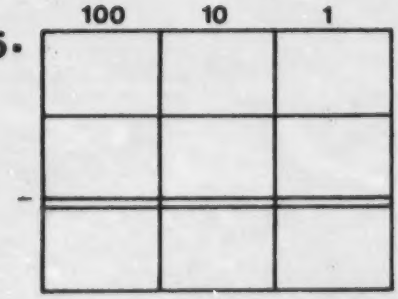
3.



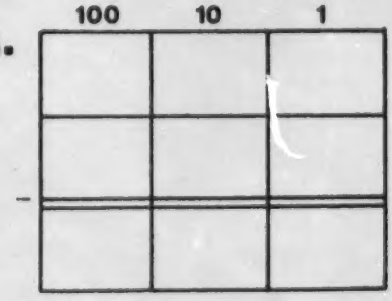
4.



5.



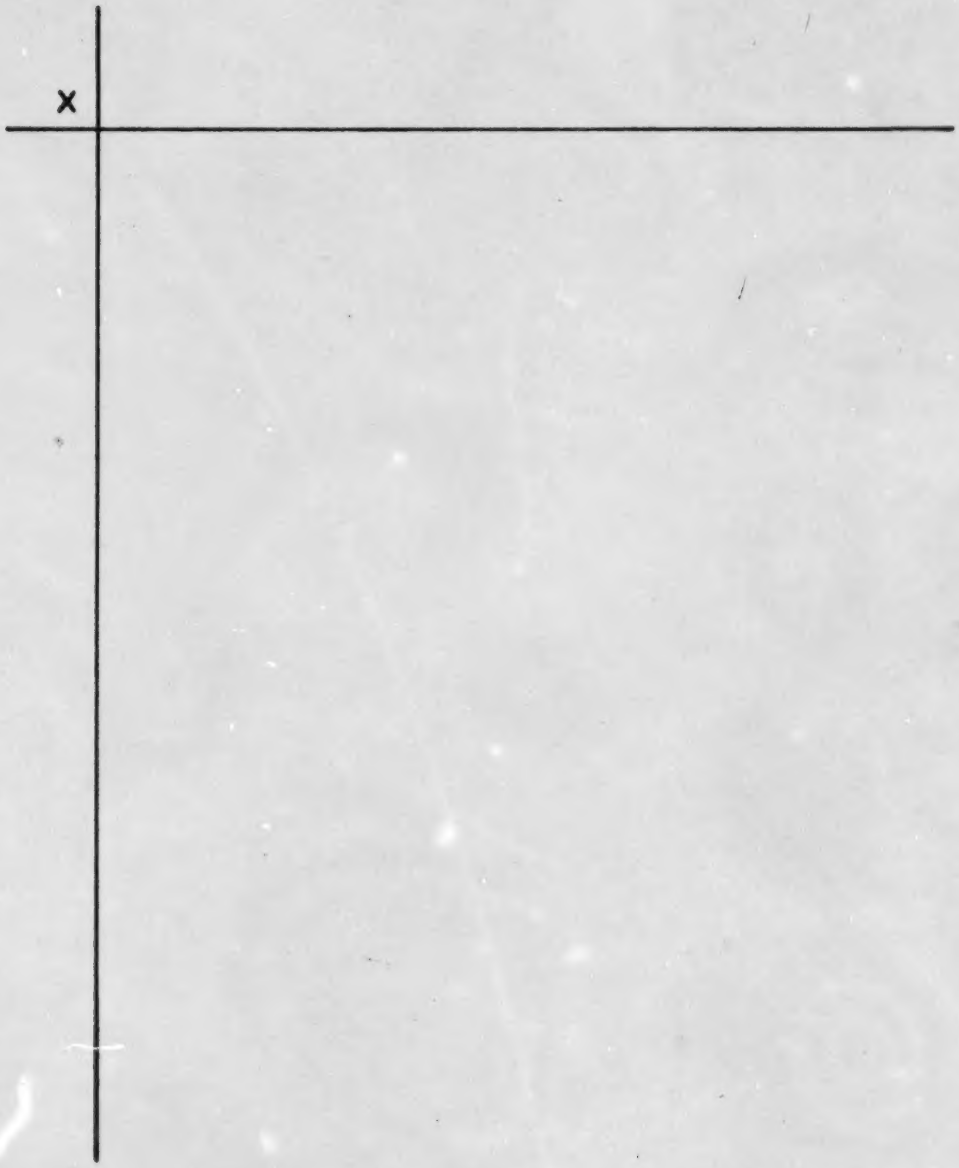
6.



MULTIPLICATION MAT

Activity Sheet 9A.

Name: _____



MULTIPLICATION

Activity Sheet 9B.

Name: _____

Make multiplication arrays using numbers 1-9. Record answers in time table below. You may need extra paper.

1. x ____ 2. x ____ 3. x ____ 4. x ____

5. x ____ 6. x ____ 7. x ____ 8. x ____

9. x ____ 10. x ____ 11. x ____ 12. x ____

x	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5						30				
6										
7										
8										
9										
10										

Example.

x | • • • • • •

• | • • • • • •
• | • • • • • •
• | • • • • • •
• | • • • • • •
• | • • • • • •

5 x 6 = 30

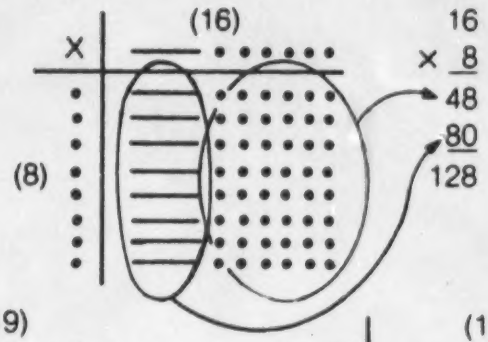
MULTIPLICATION

Activity Sheet 11A.

Name: _____

Solve these problems.

Example:



16
× 8
48
80
128

1. (19) X (6)

19
× 6
___

2. (13) X (9)

13
× 9
___

3. (17) X (5)

17
× 5
___

4. (15) X (7)

15
× 7
___

5. (16) X (4)

16
× 4
___

6. (12) X (8)

12
× 8
___

7. 13
× 7
___

8. 19
× 9
___

9. 16
× 8
___

10. 12
× 6
___

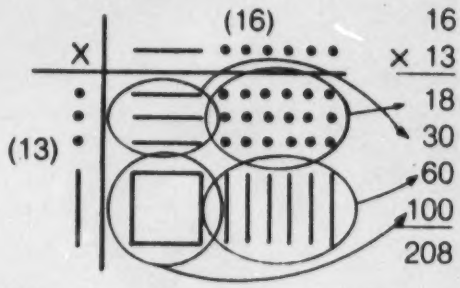
MULTIPLICATION

Activity Sheet 11B.

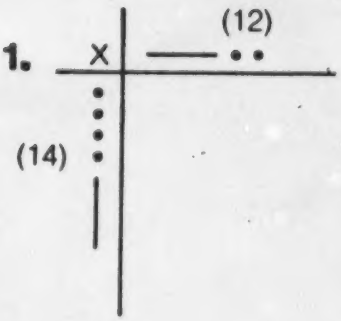
Name: _____

Solve the following problems.

Example:

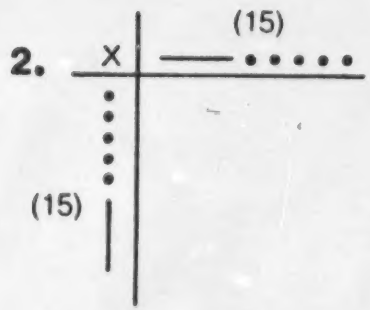


1.



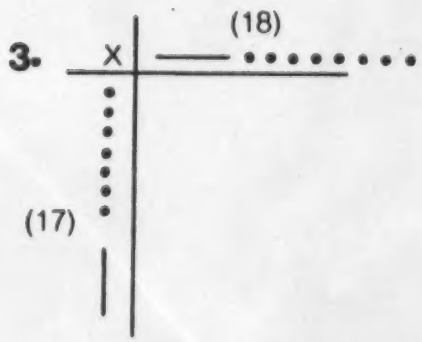
12
× 14

2.

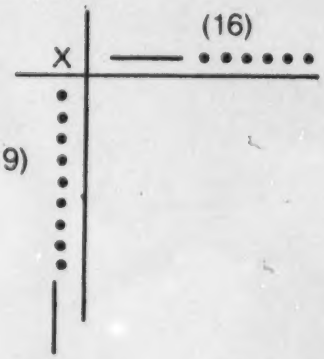


15
× 15

3.

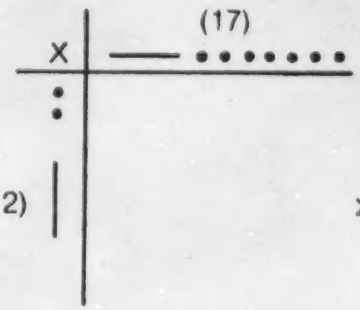


4.



16
× 19

5.



17
× 12

6.

18
× 17

13
× 19

15
× 12

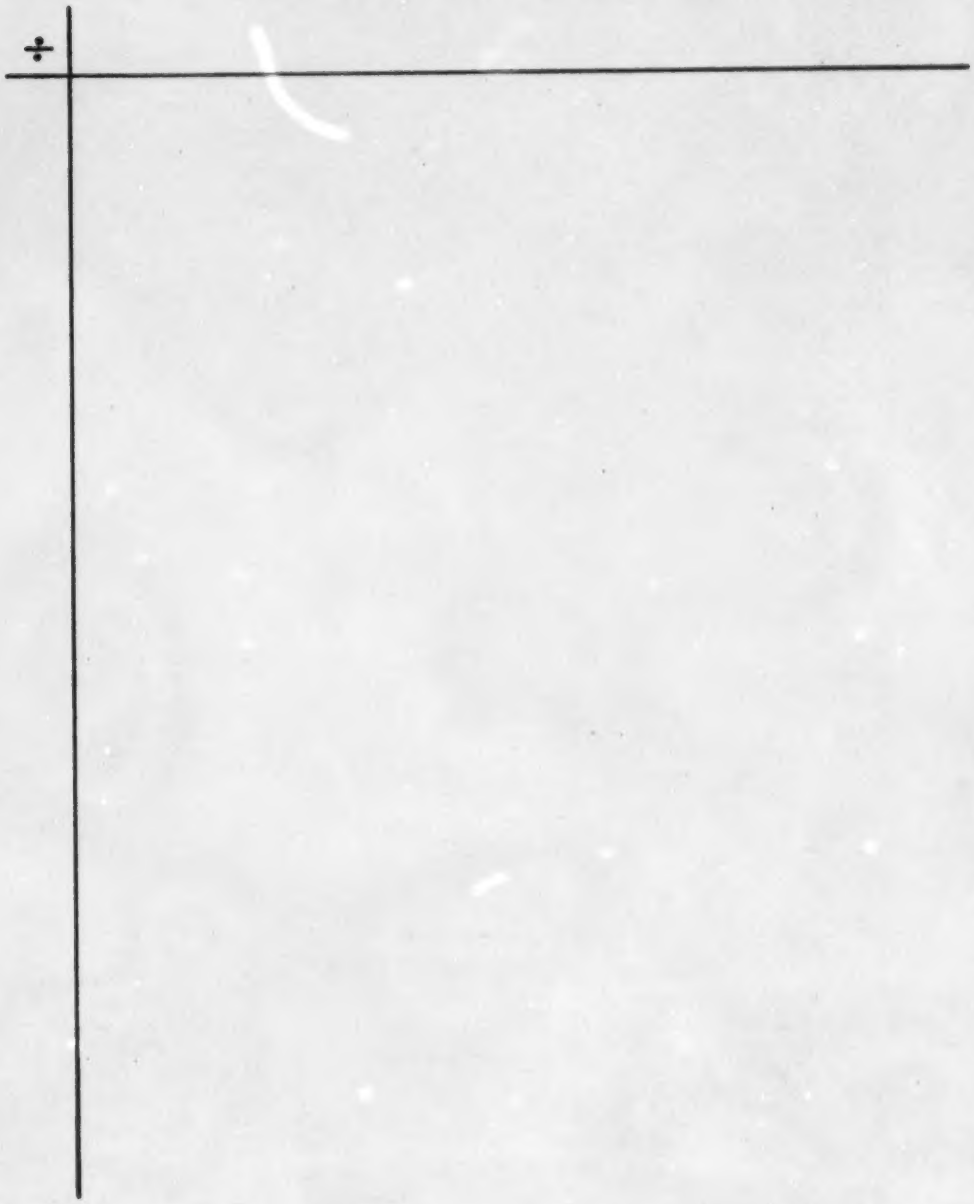
14
× 17

13
× 26

DIVISION MAT

Activity Sheet 12.

Name: _____



DIVISION

Activity Sheet 13.

Name: _____

Solve these division problems by filling in the arrays.

1. ÷ | 14 | ☐ left over

6 | • • •

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